

1-Sperner hypergraphs and new characterizations of threshold graphs

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Abstract:

We introduce a new class of hypergraphs, the class of 1-Sperner hypergraphs. A hypergraph \mathcal{H} is said to be 1-*Sperner* if every two distinct hyperedges e, f of \mathcal{H} satisfy $\min\{|e \setminus f|, |f \setminus e|\} = 1$. We prove a decomposition theorem for 1-Sperner hypergraphs and examine several of its consequences, including bounds on the size of 1-Sperner hypergraphs and a new, constructive proof of the fact that every 1-Sperner hypergraph is threshold. (A hypergraph $\mathcal{H} = (V, \mathcal{E})$ is said to be *threshold* if there exist a non-negative integer weight function $w : V \rightarrow \mathbb{Z}_+$ and a non-negative threshold $t \in \mathbb{Z}_+$ such that for every subset $X \subseteq V$, we have $\sum_{x \in X} w(x) \geq t$ if and only if X contains a hyperedge.)

A hypergraph is said to be *Sperner* (or: *a clutter*) if no hyperedge is contained in another one. Given a graph G , the *clique hypergraph* of G is the hypergraph $\mathcal{C}(G)$ with vertex set $V(G)$ in which the hyperedges are exactly the maximal cliques of G . The clique hypergraphs of graphs are known to be exactly those Sperner hypergraphs \mathcal{H} that are also *normal* (or: *conformal*), that is, for every set $X \subseteq V(\mathcal{H})$ such that every pair of elements in X is contained in a hyperedge, there exists a hyperedge containing X . We show that within the class of normal Sperner hypergraphs, the (generally properly nested) classes of 1-Sperner hypergraphs, of threshold hypergraphs, and of 2-asummable hypergraphs coincide. This yields new characterizations of the class of threshold graphs.

The talk is based on [1].¹

References

- [1] E. Boros, V. Gurvich, and M. Milanič. 1-Sperner hypergraphs. Manuscript, 2015.

¹Remark: The manuscript should be soon (hopefully before GROW) posted on arXiv, in which case the reference can be updated accordingly.